SENSITIVITY OF A COMPUTATIONAL VERSION OF THE KIRCHHOFF INTEGRAL THEOREM TO SURFACE DISCRETIZATION

by

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INTRODUCTION

- What is the Kirchhoff Integral Theorem?
- Statement of Purpose—Motivating Example
- Background
- What is the K.I.T. mathematically?
- Overview of Methodology

BACKGROUND REFERENCES

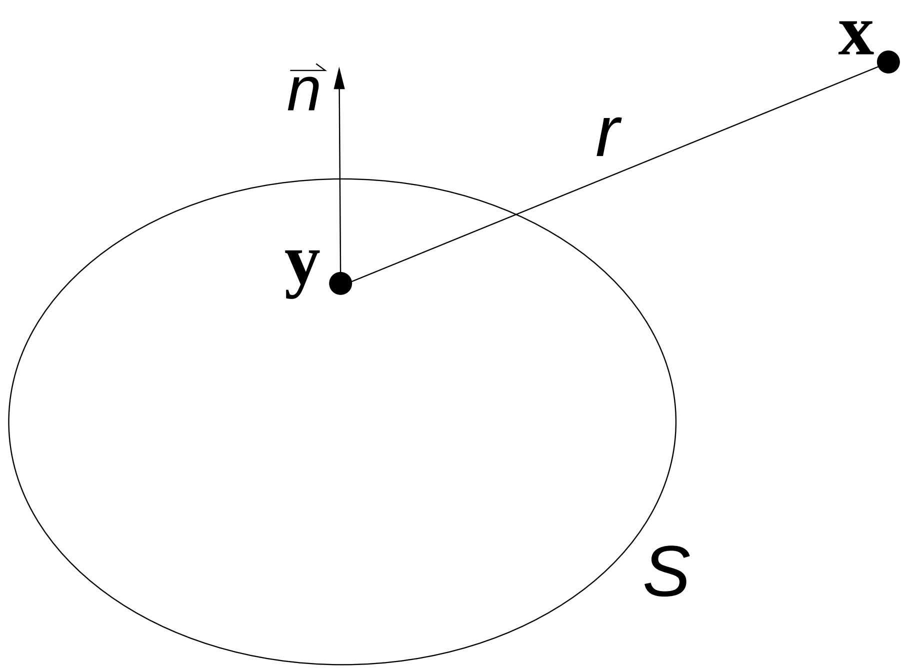
G. R. Kirchhoff, "Zur Theorie der Lichtstrahlen," Annalen der Physik und Chemie, **18**:663–695 (1883)

A. S. Lyrintzis, "Review: The Use of Kirchhoff's Method in Computational Aeroacoustics," Journal of Fluids Engineering **116**, pp. 665–675 (1994)

A. D. Pierce, *Acoustics: An Introduction to Its Physical Principles and Applications*, (Acoustical Society of America, 1989)

J. A. Stratton, *Electromagnetic Theory*, (McGraw-Hill, 1941)

MATHEMATICS OF THE K.I.T.



$$\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} - \nabla^2\Phi = 0$$

$$\tau = t - \frac{r}{c}$$

$$p\left(\mathbf{x}, t\right) = \frac{1}{4\pi}\int_S \left[\frac{p}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p}{\partial n} + \frac{1}{c\,r}\frac{\partial r}{\partial n}\frac{\partial p}{\partial \tau}\right]_\tau dS$$

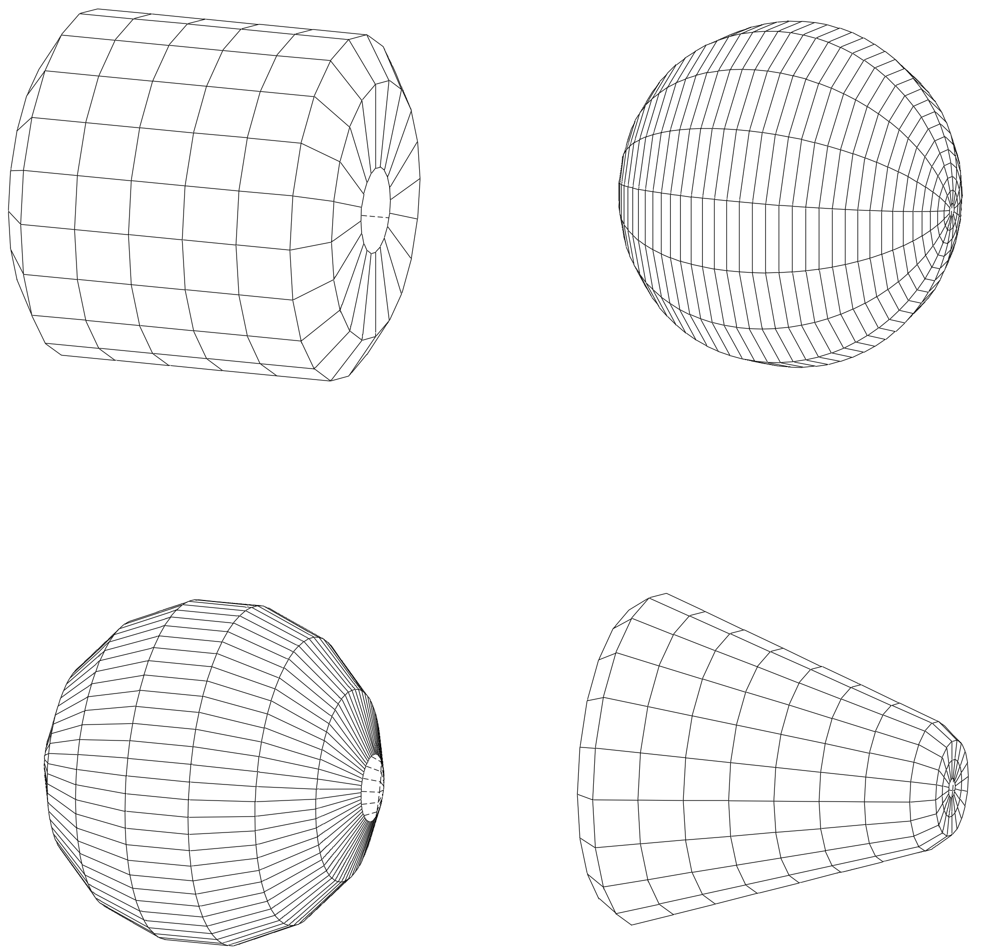
OVERVIEW OF METHODOLOGY

- Computational Formulation
- Validation
- The Kirchhoff Grid
- Conclusions and Further Research

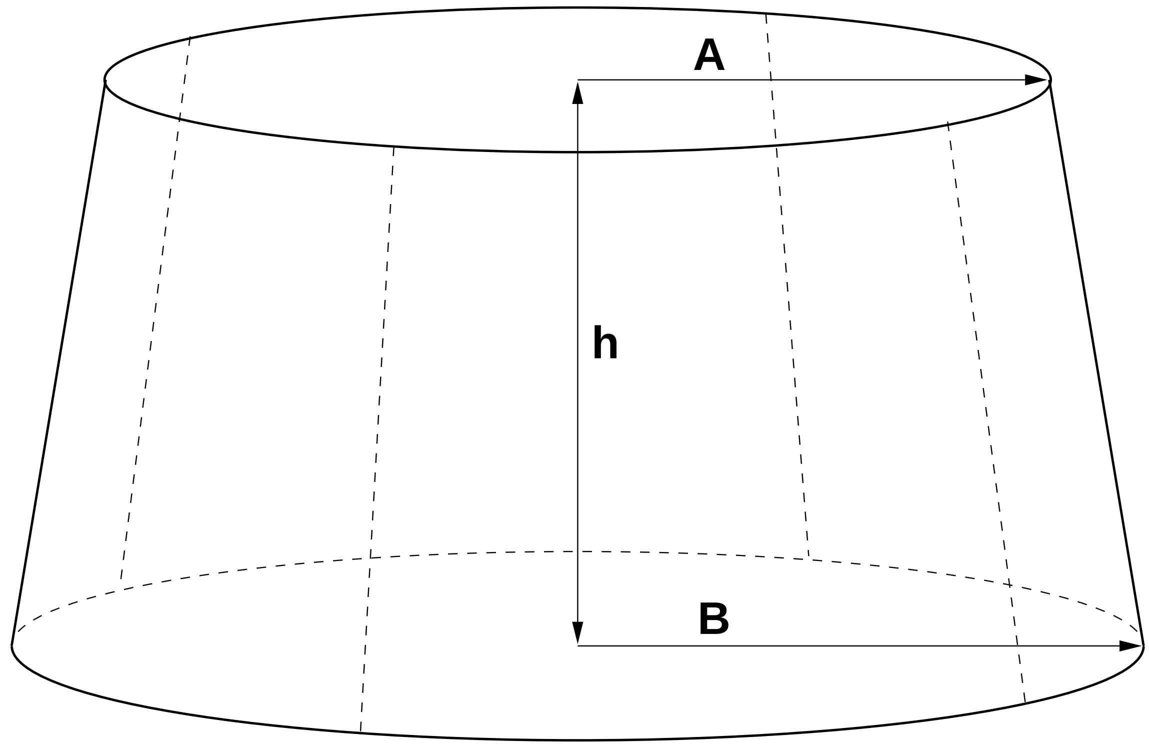
COMPUTATIONAL FORMULATION

- Computational Grid Formulation
- (Pressure Calculated Analytically)
- Analytical-Numerical Formulation
- Fully Numerical Formulation
- Fourier Algorithm

EXAMPLE COMPUTATIONAL GRIDS



AREA PATCHES FOR SURFACE INTEGRATION



$$dS = \frac{2\pi R_f h}{n_{cr}} \qquad R_f = \frac{A+B}{2} \qquad n_{cr} = 4$$

ANALYTICAL EXPRESSIONS FOR KIRCHHOFF TERMS

$$p = \frac{A}{R} \cos\left(2\theta\right) e^{i(kR - \omega t)}$$

$$\frac{\partial p}{\partial n} = \frac{A}{R} \cos\left(2\theta\right) e^{i(kR - \omega t)} \left(\frac{ikR - 1}{R^2}\right)$$

$$\frac{\partial p}{\partial t} = \frac{-iA\omega}{R} \cos\left(2\theta\right) e^{i(kR - \omega t)}$$

$$\frac{\partial r}{\partial n} = \cos\left(\alpha\right) = \frac{\vec{r} \cdot \vec{n}}{\|\vec{r}\|\|\vec{n}\|}$$

NUMERICAL EXPRESSIONS FOR KIRCHHOFF TERMS

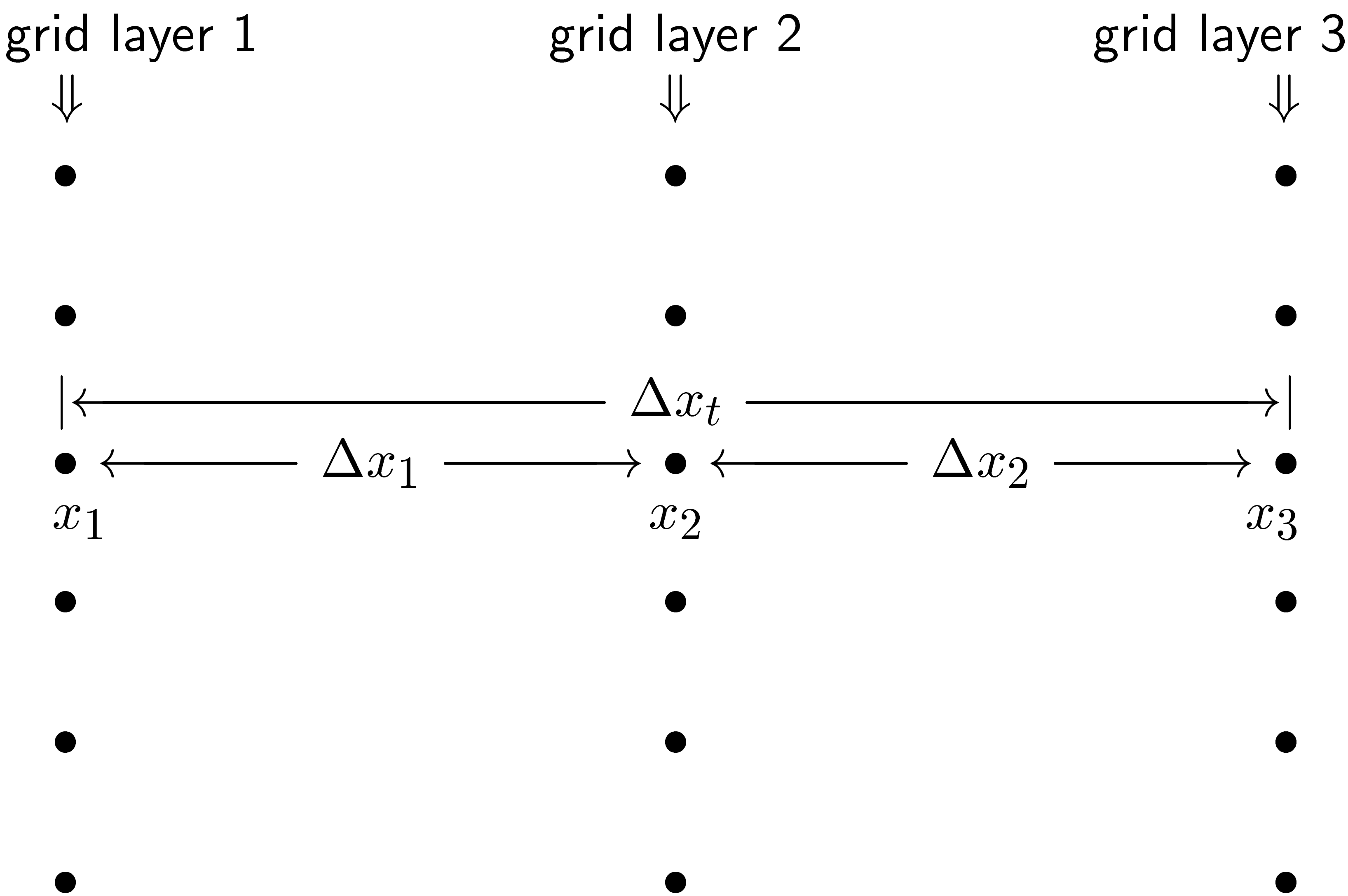
$$\frac{\partial p}{\partial t} = \frac{p_{t+\Delta t} - p_{t-\Delta t}}{2\Delta t}$$

$$\frac{\partial f(x)}{\partial x} = \frac{\left(-2\Delta x_1 \Delta x_2 - \Delta x_2^2\right) f(x_1) + (\Delta x_1 + \Delta x_2)^2 f(x_2) - \Delta x_1^2 f(x_3)}{\Delta x_1 \Delta x_2 (\Delta x_1 + \Delta x_2)}$$

$$\frac{\partial p}{\partial n} = \frac{\partial f(x)}{\partial x}\frac{\partial x}{\partial n} = \frac{\partial f(x)}{\partial x} \quad \text{where } f(x) = p(x)$$

$$\frac{\partial r}{\partial n} = \frac{\partial f(x)}{\partial x}\frac{\partial x}{\partial n} = \frac{\partial f(x)}{\partial x} \quad \text{where } f(x) = r(x)$$

POINTS IN GRID LAYERS

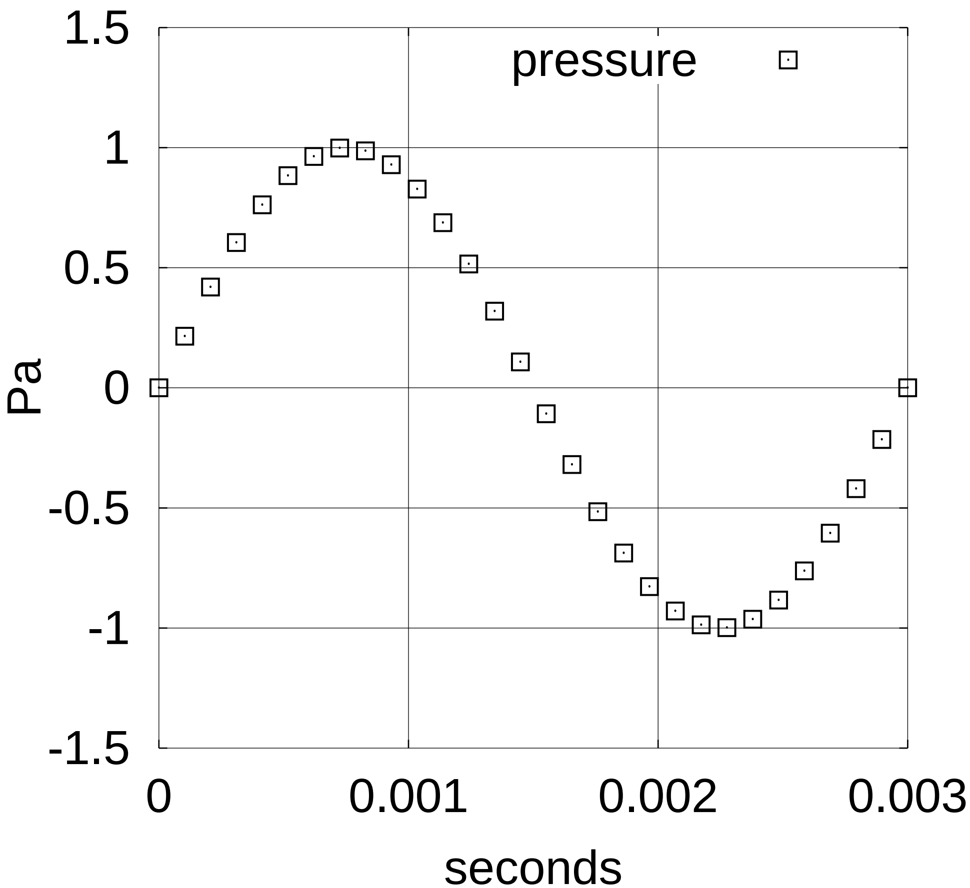


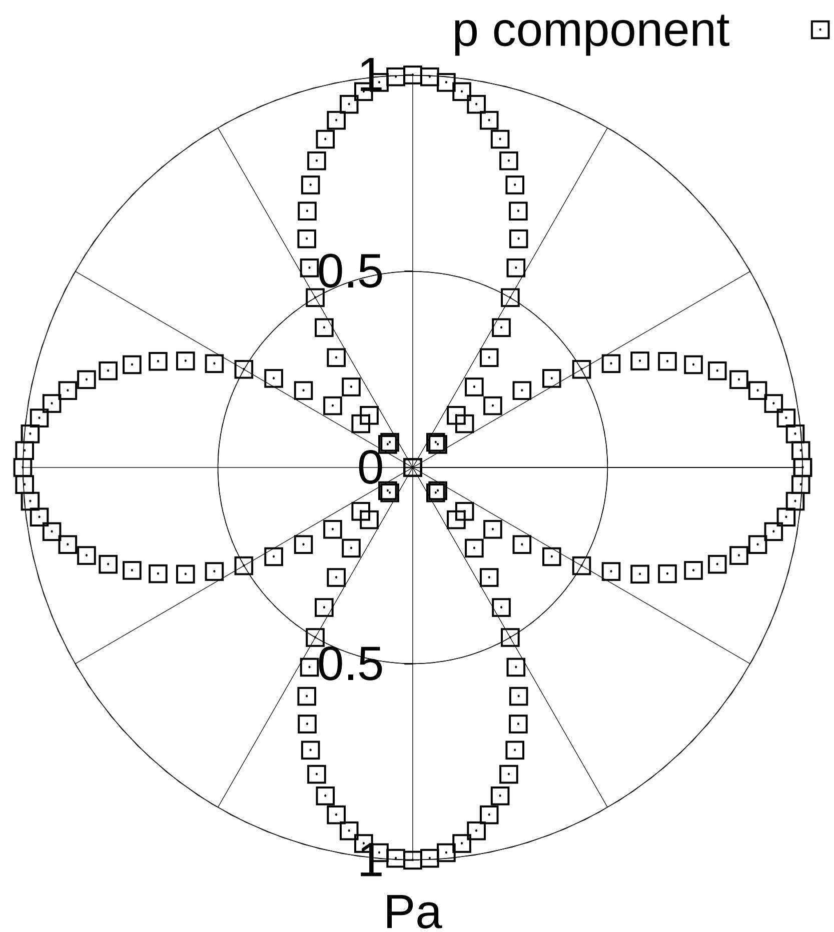
DISCRETE TIME FOURIER TRANSFORM

$$F_c = \sum_{m=1}^{n_t} p\,(m)\, e^{i\omega_c t}\, \Delta t$$

provides the Fourier component of pressure at one point in space for a single frequency, ω_c.

APPROXIMATION OF CONTINUITY

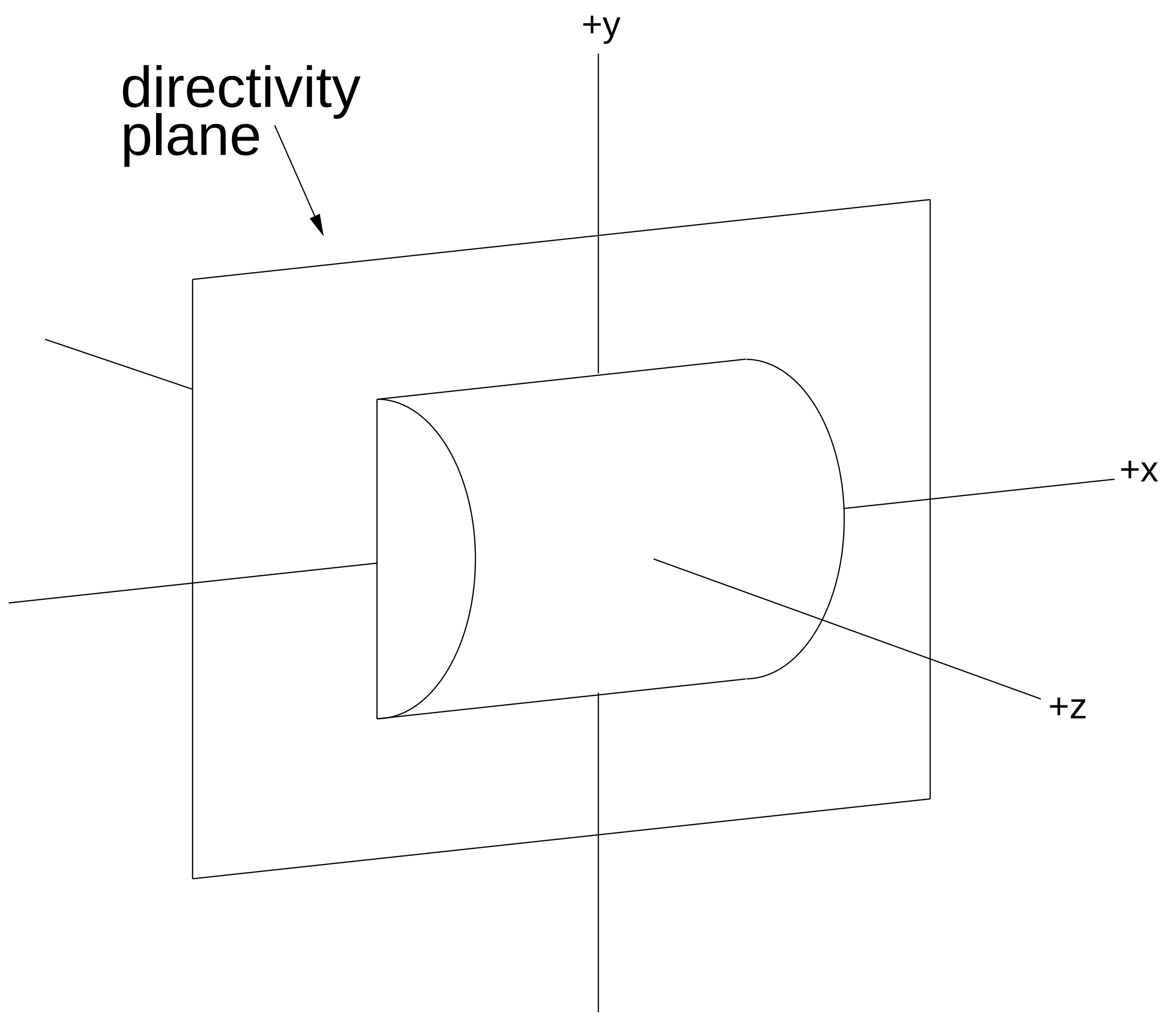




VALIDATION

- Error Definitions
- Analytical-Numerical
- Fully Numerical
- Comparison
- Frequency Separation

DIRECTIVITY PLANE

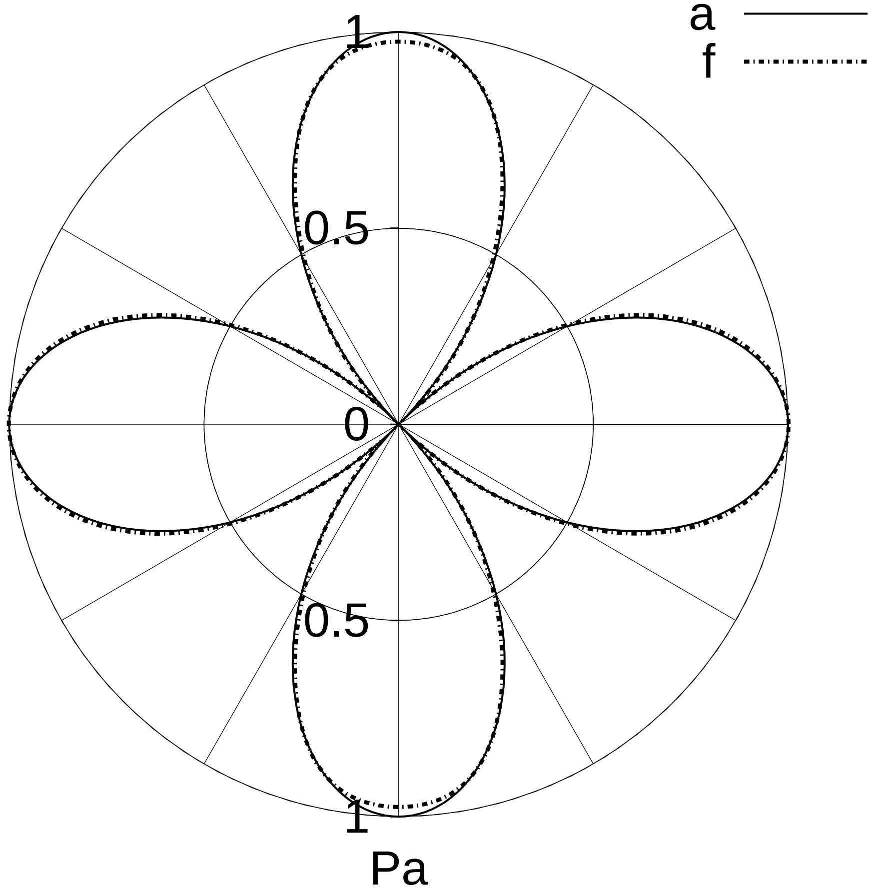


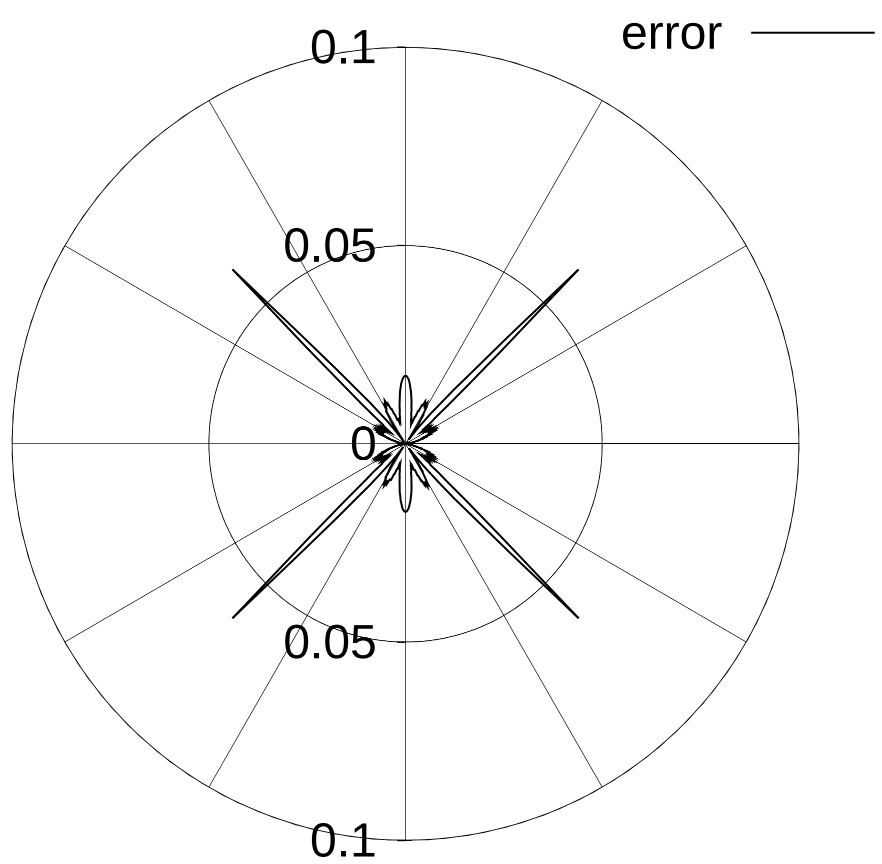
ERROR DEFINITIONS

$$\text{error :} \quad \frac{|p_f - p_a|}{\max p_a}$$

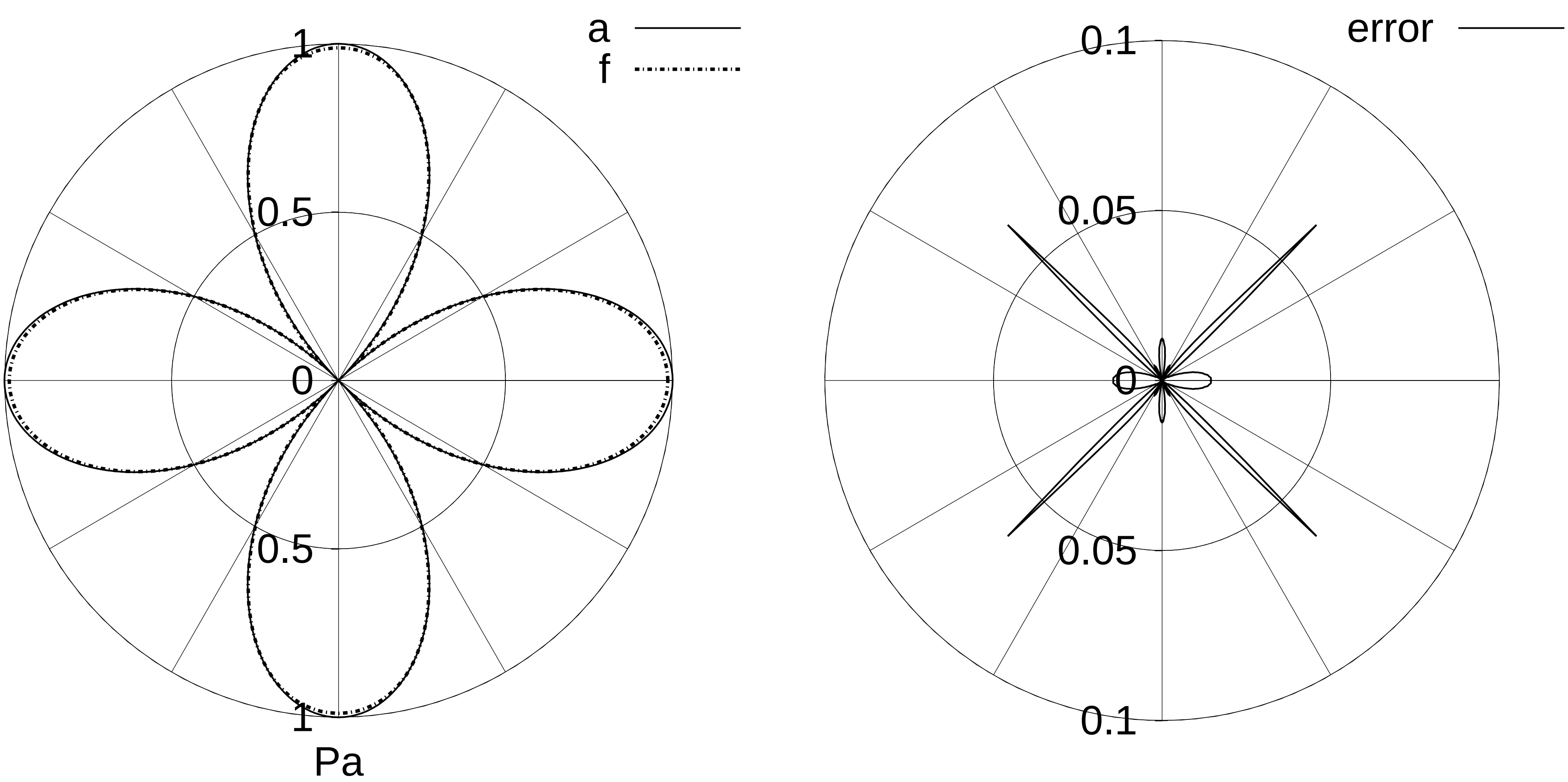
$$\text{local error :} \quad \frac{|p_f - p_a|}{\max p_a}\frac{p_a}{\max p_a}$$

ANALYTICAL-NUMERICAL VALIDATION

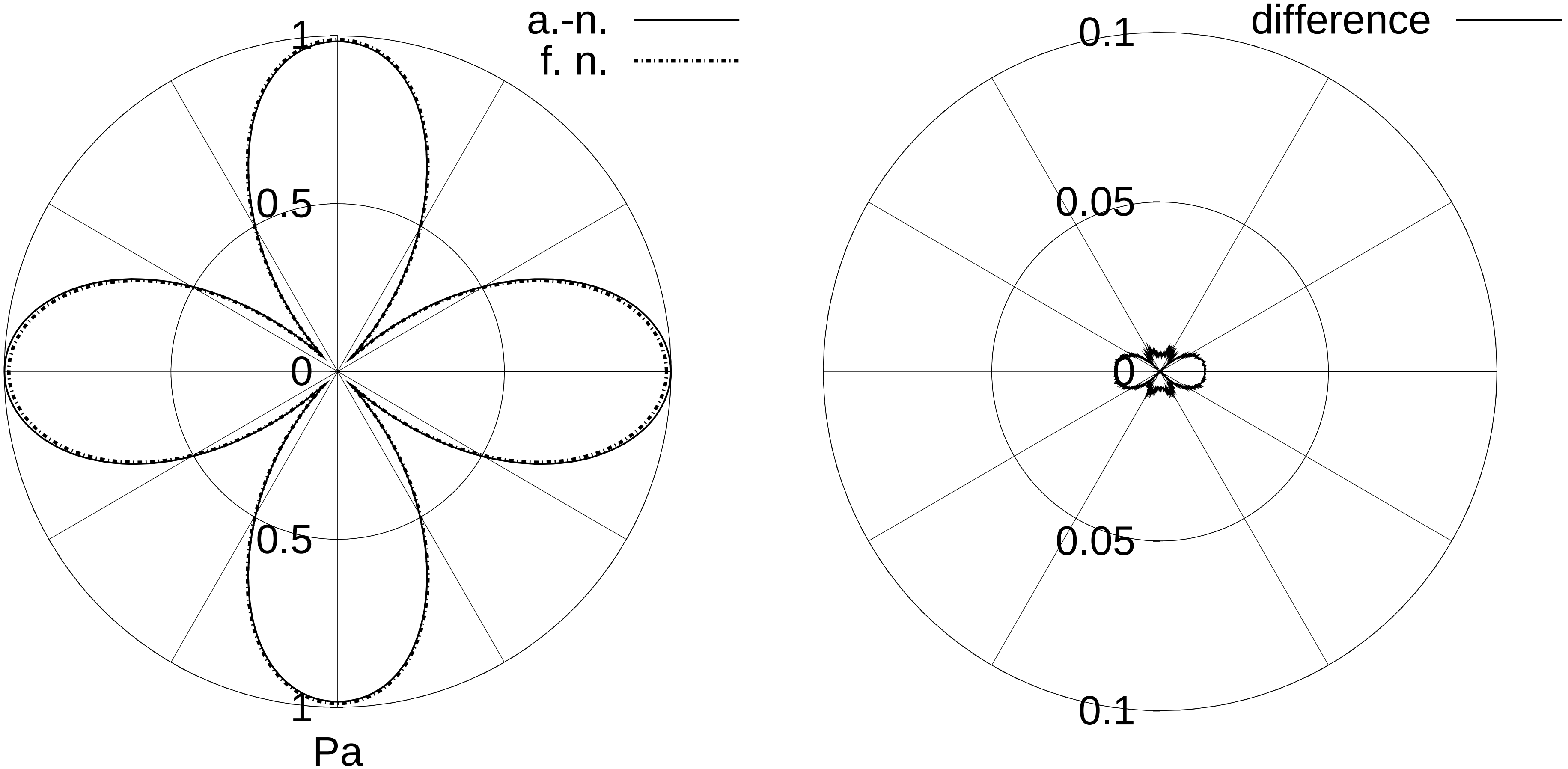




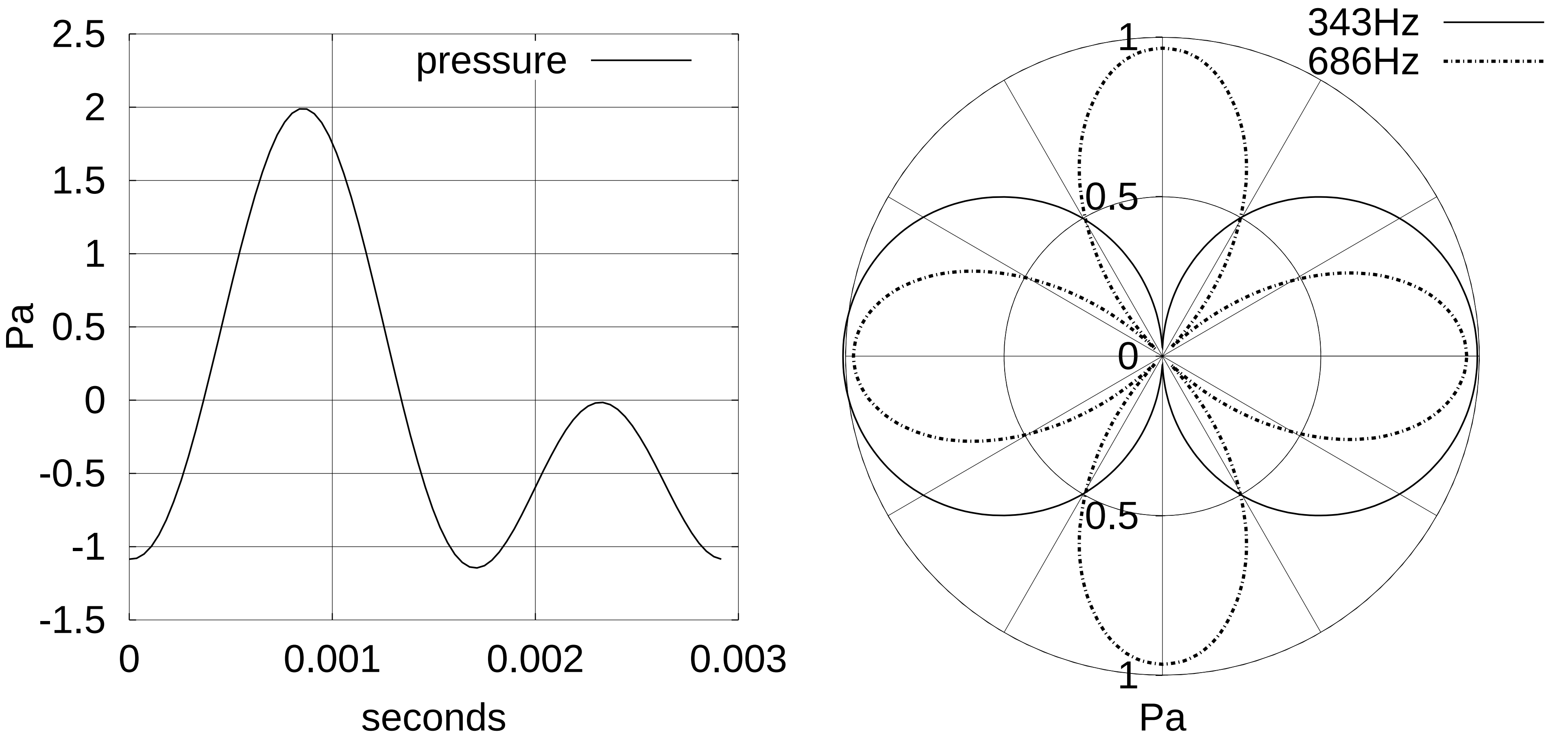
FULLY NUMERICAL VALIDATION



COMPARISON OF THE FORMULATIONS



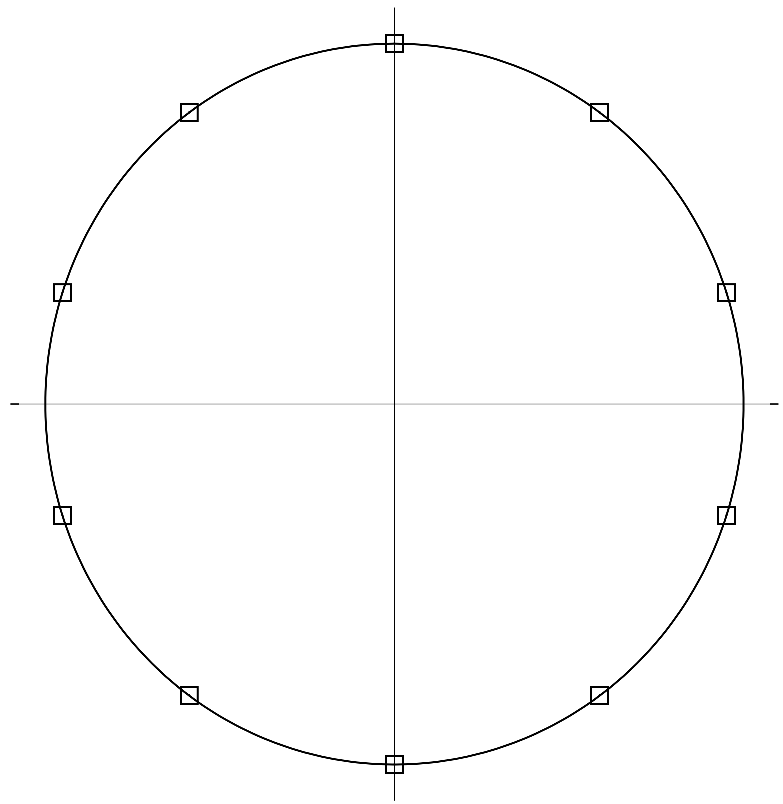
FREQUENCY SEPARATION



THE KIRCHHOFF GRID

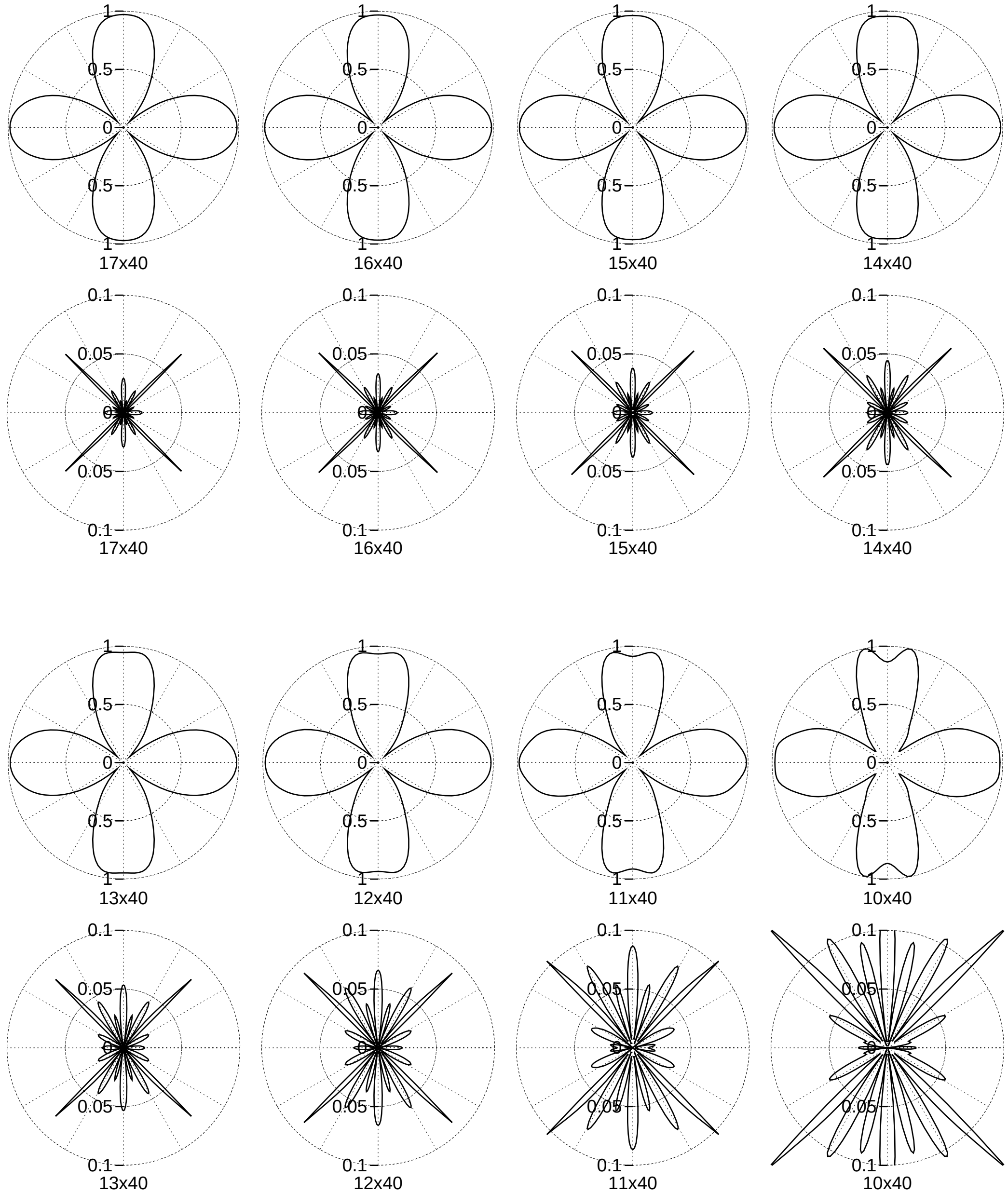
- Grid Point Spacing
- Grid Layer Separation
- Time Sampling
- Size of the Computational Grid
- Grid Shape

AXIAL GRID POINT SPACING

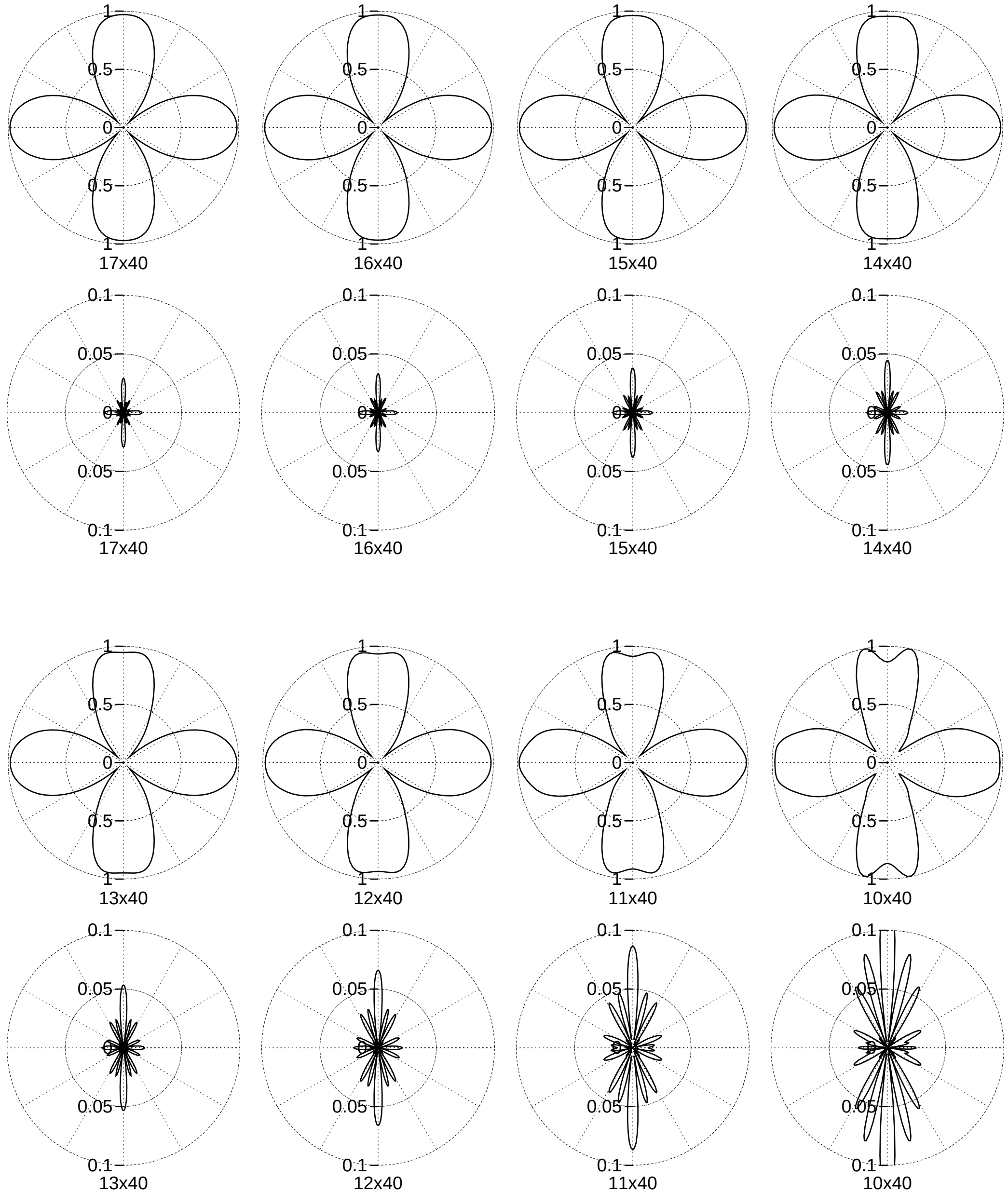


$$d_{ax} = \frac{\pi r_s}{n_{ax}} \qquad n_{ax} = 5$$

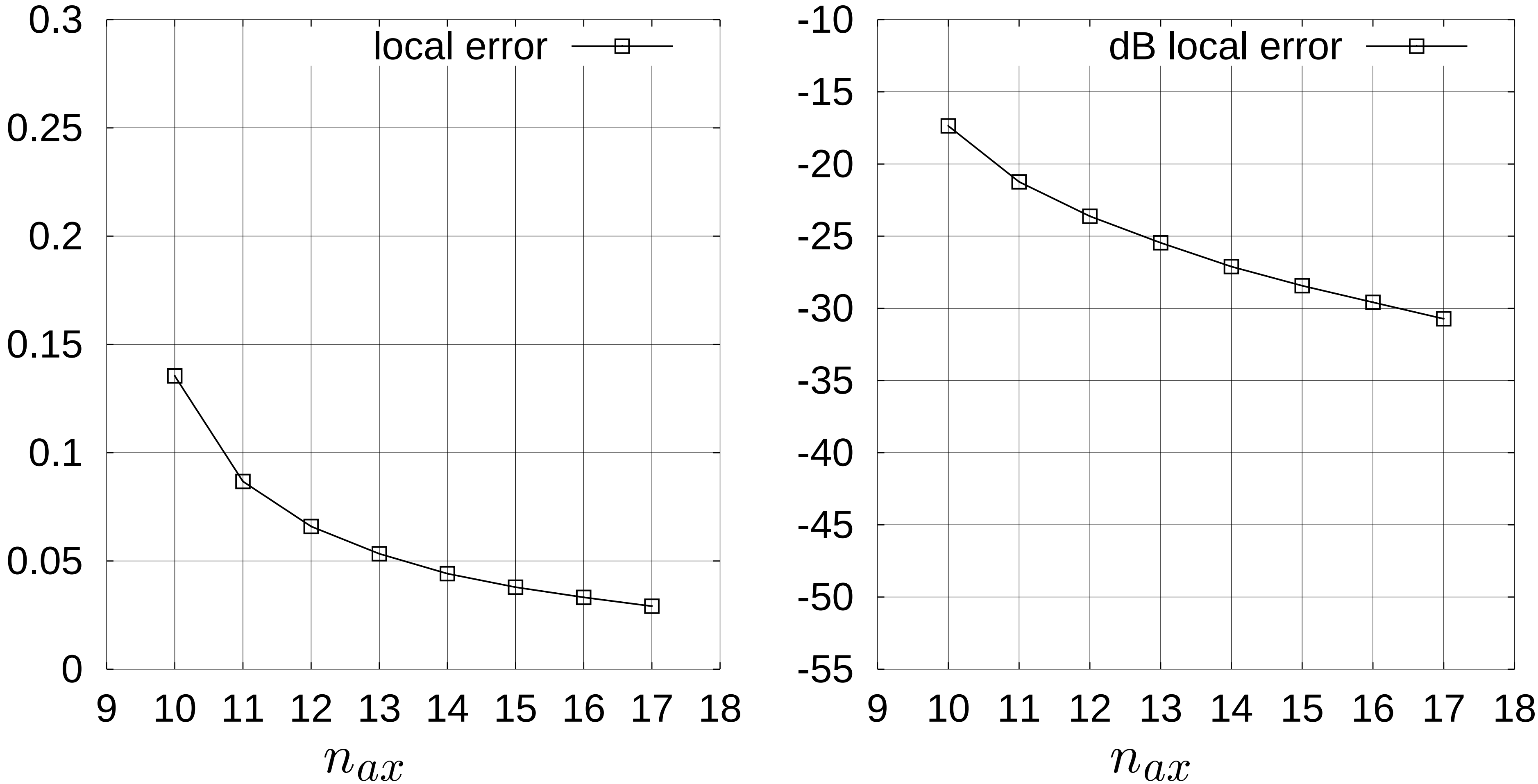
INCREASING AXIAL GRID POINT SPACING (error)



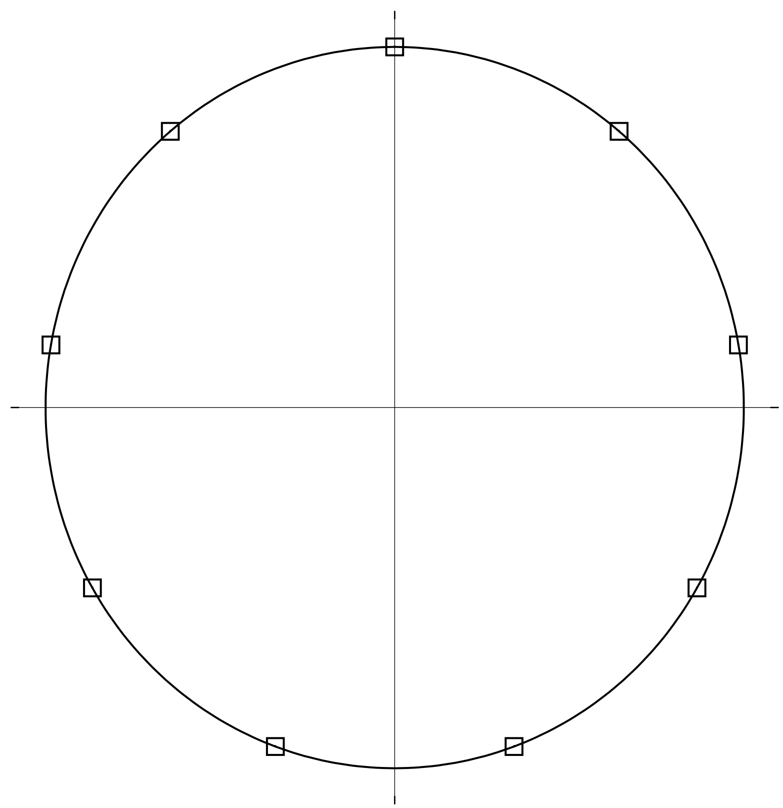
INCREASING AXIAL GRID POINT SPACING (local error)



AXIAL GRID POINT SPACING VS. LOCAL ERROR

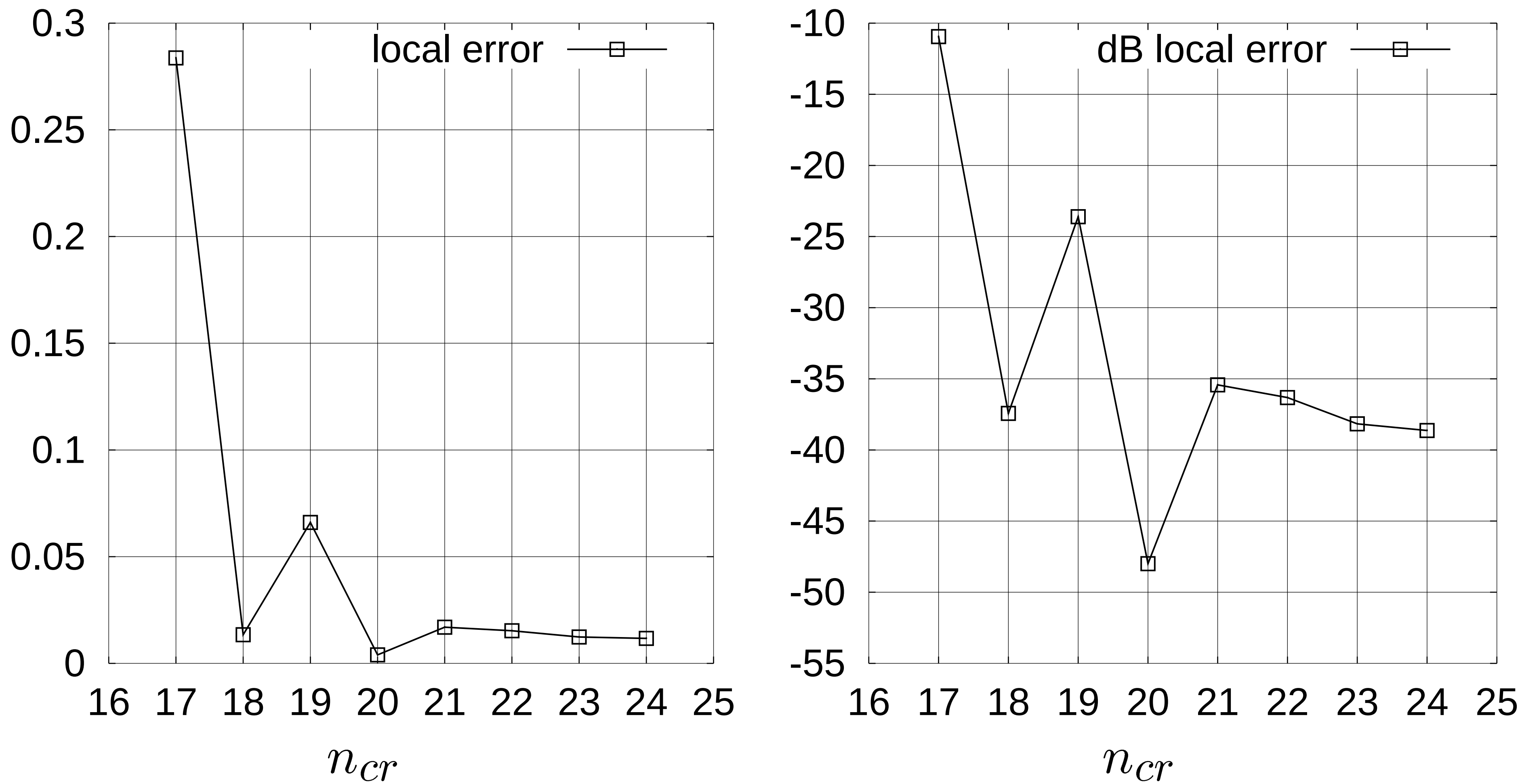


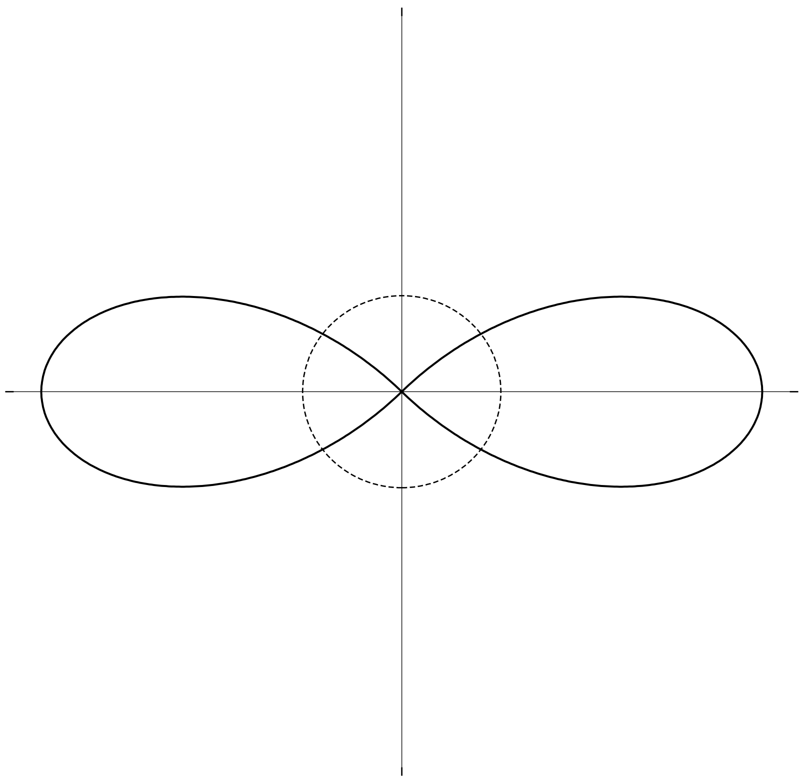
CIRCUMFERENTIAL GRID POINT SPACING



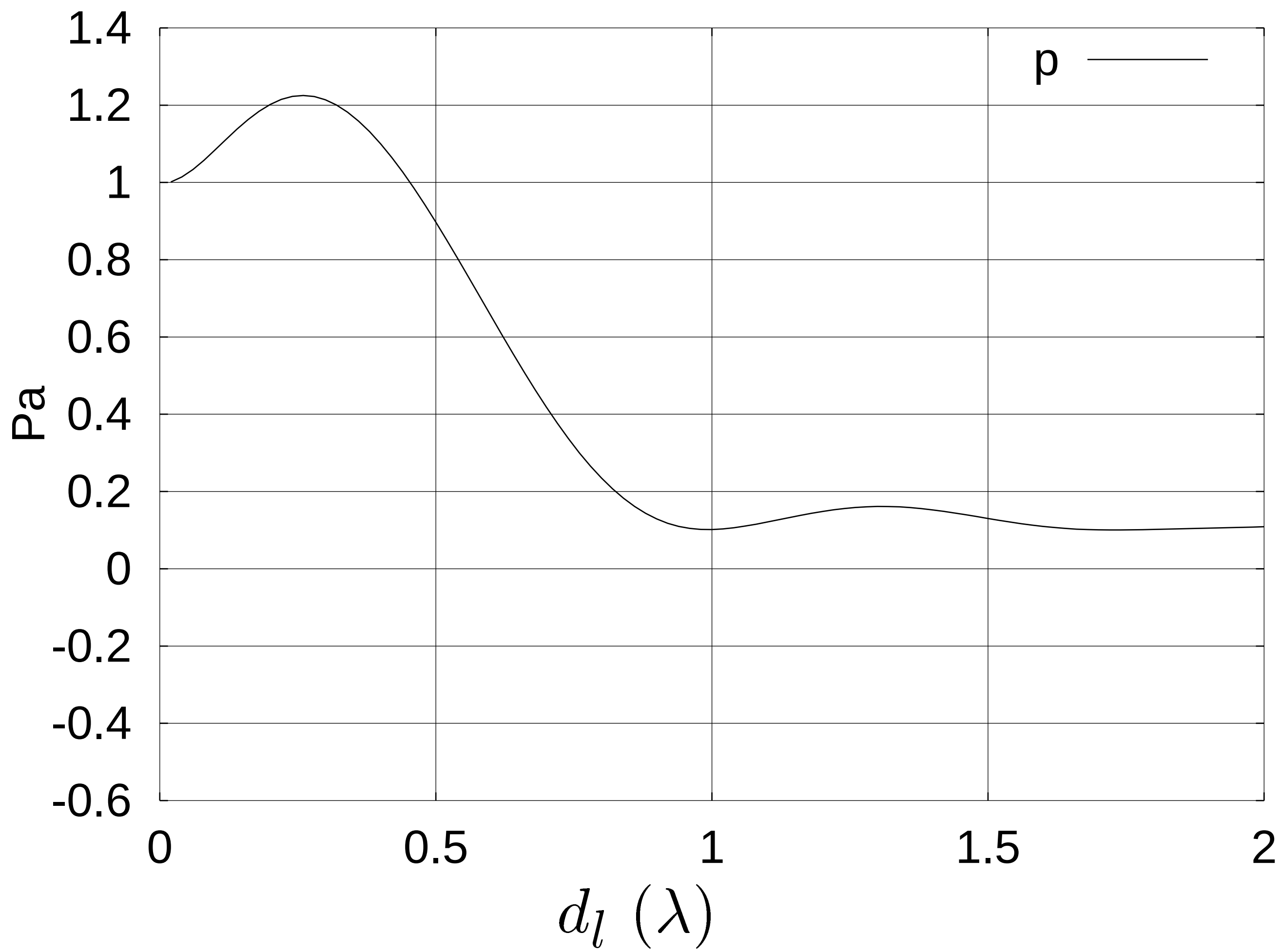
$$d_{cr} = \frac{2\pi r_x}{n_{cr}} \qquad n_{cr} = 9$$

CIRCUM. GRID POINT SPACING VS. LOCAL ERROR

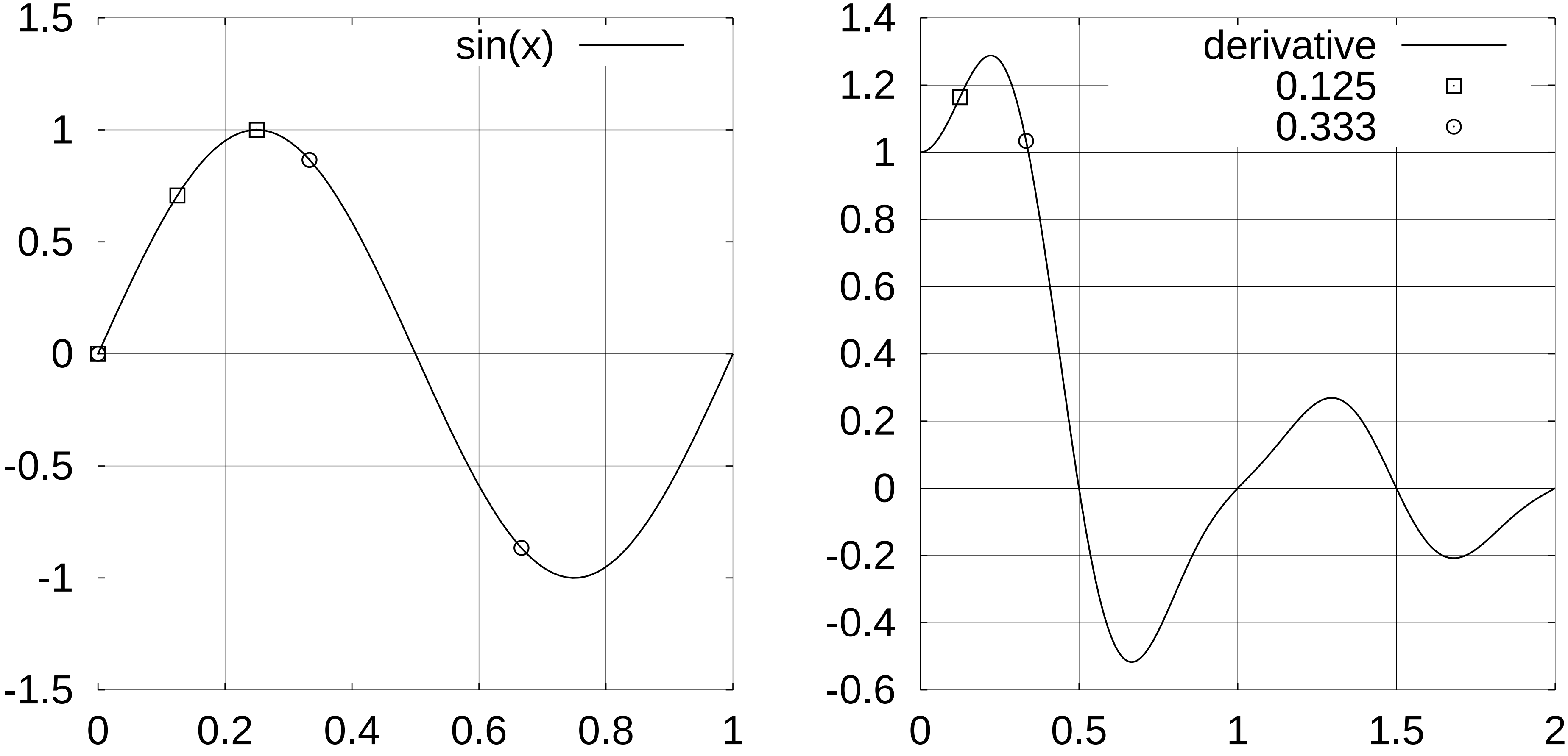




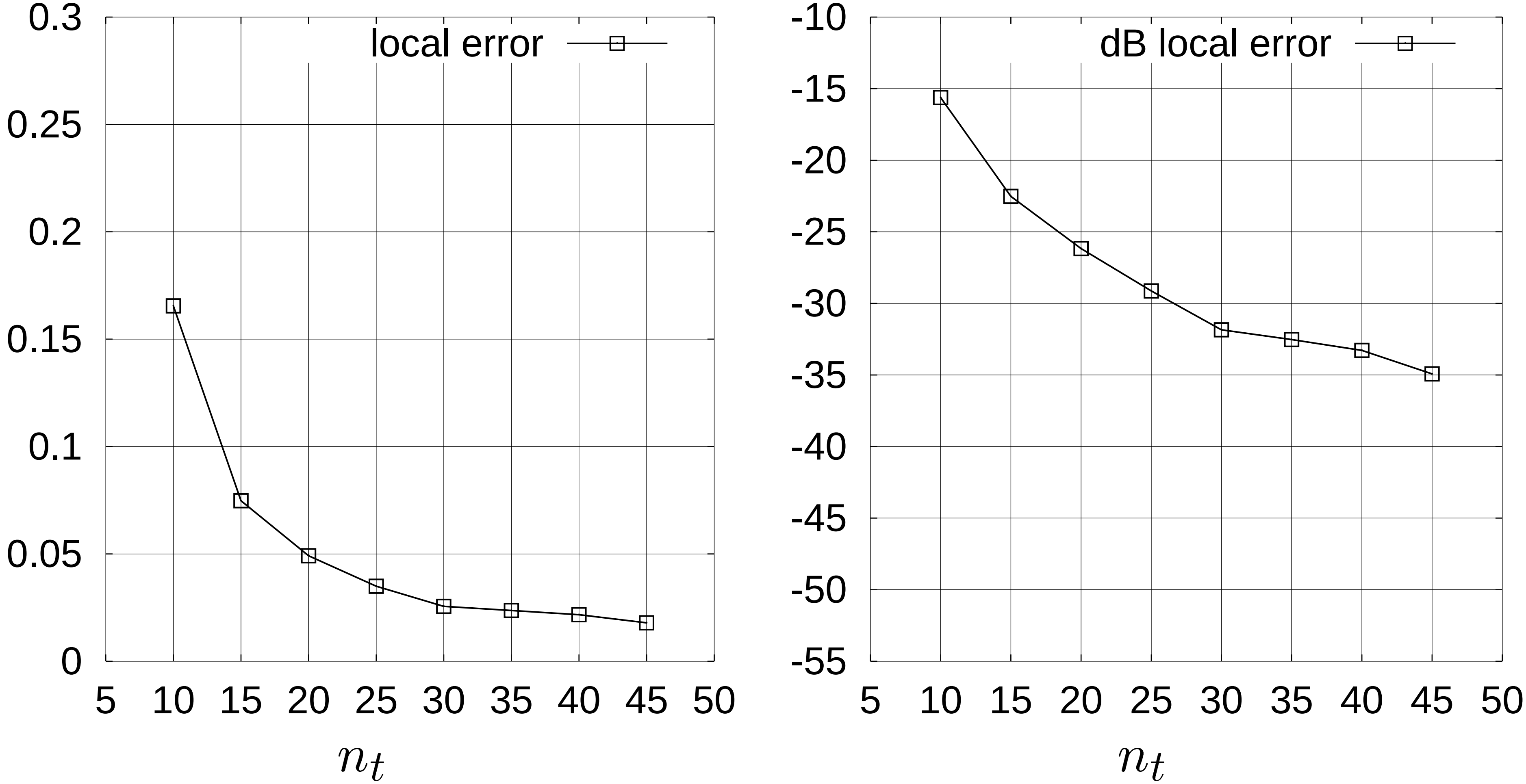
GRID LAYER SEPARATION



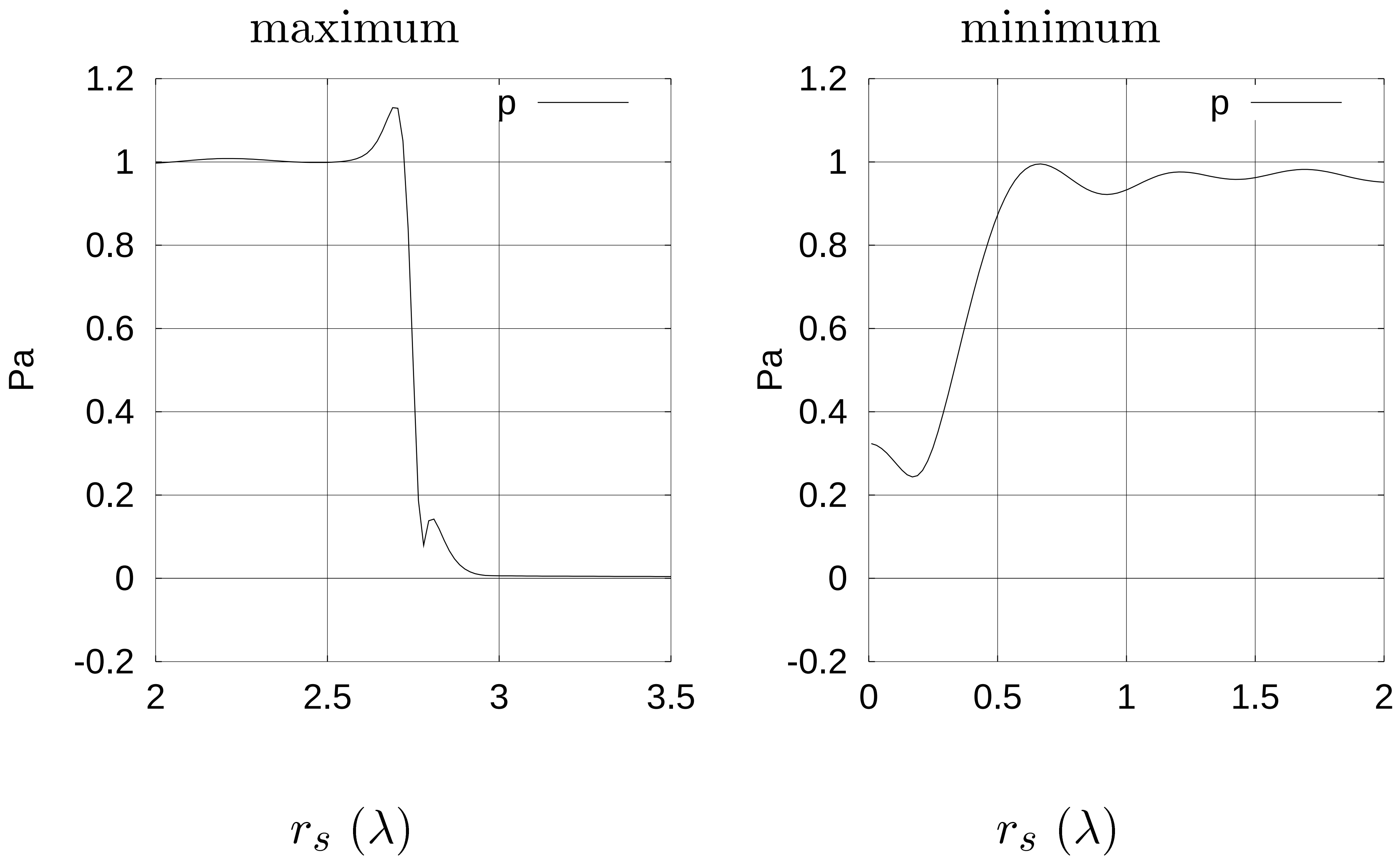
2ND ORDER FORWARD DIFFERENCE SCHEME



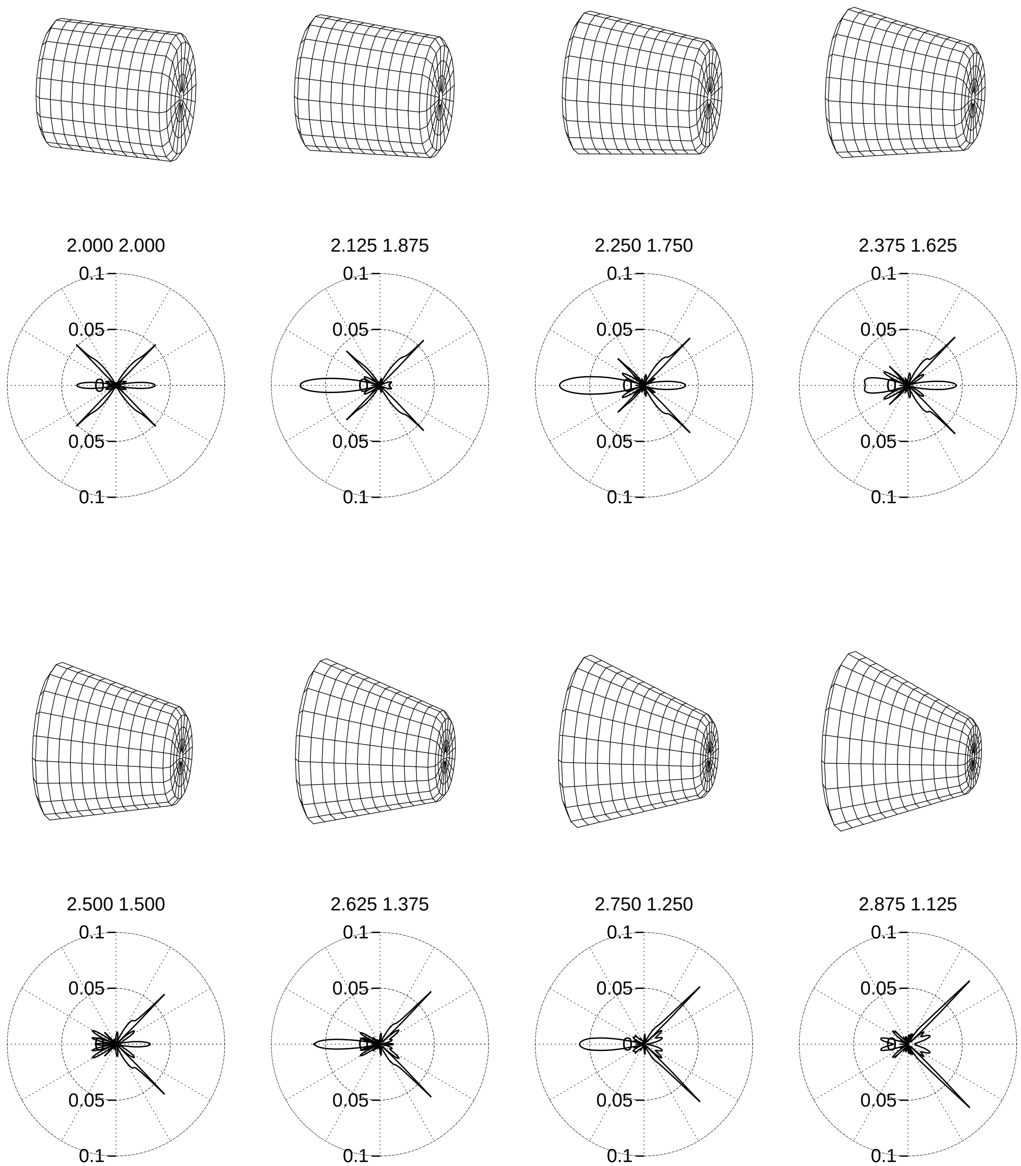
TIME SAMPLES VS. LOCAL ERROR



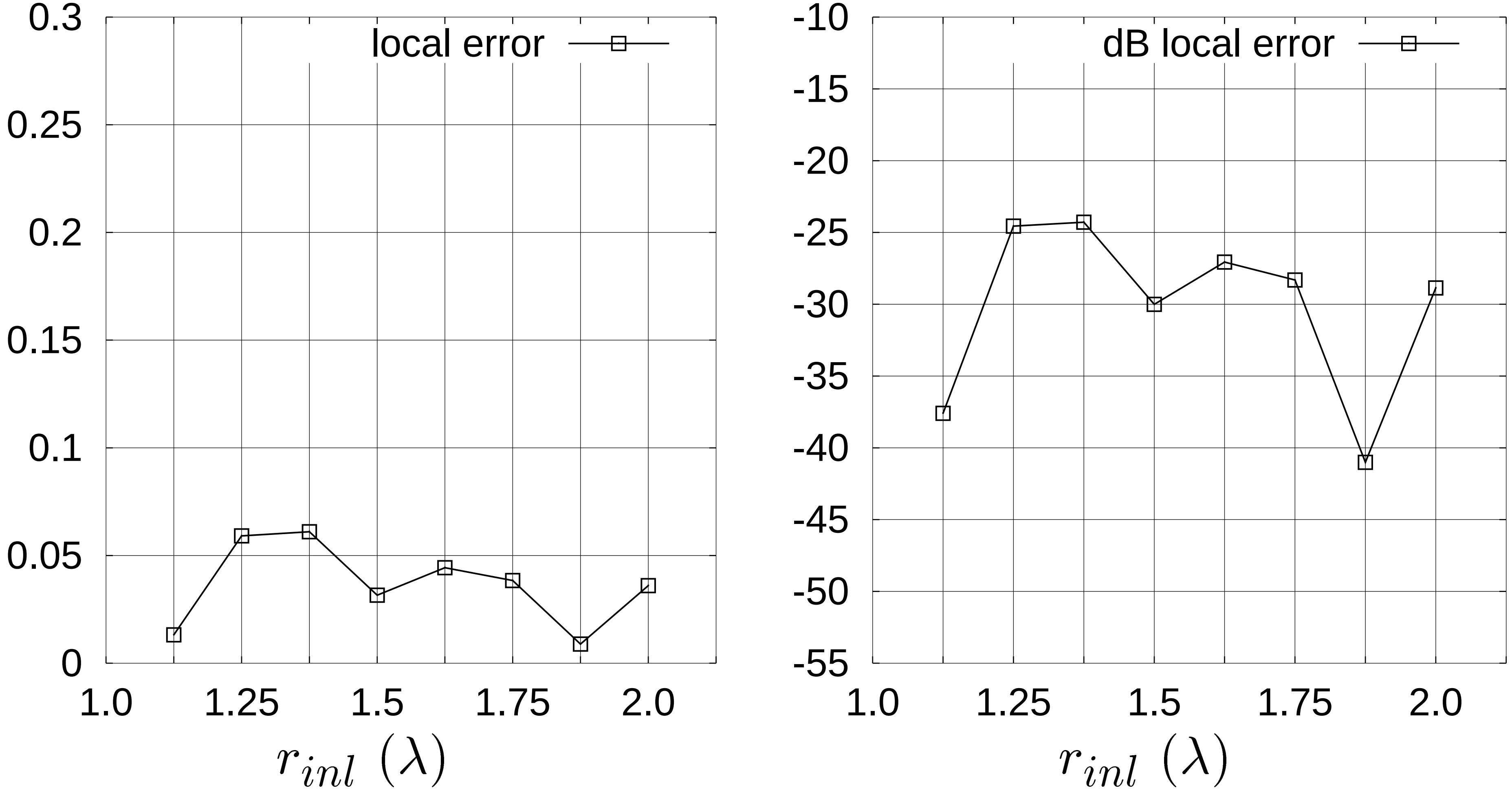
GRID SIZE LIMITS



GRID SHAPE


2.000 2.000
2.125 1.875
2.250 1.750
2.375 1.625
2.500 1.500
2.625 1.375
2.750 1.250
2.875 1.125
0.1
0.05
0
0.05
0.1

GRID SHAPE VS. LOCAL ERROR



CONCLUSIONS

- Modern Computing Resources
- Computational Formulations
- Validation
- Grid Parameters

and **FURTHER RESEARCH**

- Noise Pollution Issues
- Enhance Model with Flow (Including Supersonic)
- Pressure Variations
- Virtual "Walk" Around a Sound Source

One moment please. . .